

Density-dependent symmetry energy at subsaturation densities from nuclear mass differences

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We extract the mass-dependent symmetry energy coefficients $a_{\text{sym}}(A)$ with the nuclear mass differences reducing the uncertainties as far as possible. The estimated $a_{\text{sym}}(A)$ of ^{208}Pb is 22.4 ± 0.3 MeV, which is further used to analyze the density-dependent nuclear matter symmetry energy at subsaturation densities. The slope parameter of the symmetry energy at the saturation density ρ_0 is $L = 50.0 \pm 15.5$ MeV. Furthermore, it is found that, at the density of $\rho = 0.69\rho_0 = 0.11\text{fm}^{-3}$, the symmetry energy $S(\rho = 0.11\text{fm}^{-3}) = 25.98 \pm 0.01$ MeV and the correspondingly slope parameter is $L = 49.6 \pm 6.2$ MeV, which are consistent with other independent analysis.

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The equation of state (EOS) of isospin asymmetric nuclear matter is an active research field at present because of its importance in nuclear physics and in particular in astrophysics. Unfortunately, the variation of the EOS with respect to baryon density is still being intensely debated, especially the symmetry energy which characterizes its isospin-dependence. The density-dependent symmetry energy plays a crucial role in understanding a variety of issues in nuclear physics and astrophysics, such as the heavy ion reactions [1–4], the stability of superheavy nuclei [5], the structures, composition and cooling of neutron stars [6–9]. Because of its great importance, many authors concentrate on this issue within many independent approaches, such as the heavy ion collision, microscopic and phenomenological nuclear many body theories and collective excitations. At current one grasps some basic knowledge about the symmetry energy at low densities, while at high densities one almost know nothing even its variation tendency as the density. The slope parameter L governing the density dependence of $S(\rho)$ around the saturation density ρ_0 , has been found to correlate linearly with the neutron skin thickness of heavy nuclei such as ^{208}Pb [10–12]. Therefore, a measurement of ΔR_{np} with a high accuracy is a strong constraint of the density dependence of symmetry energy at subnormal densities. Due to the large uncertainties in measured neutron skin thickness, this has not been possible now. However, we may constrain the symmetry energy effectively with the help of other approaches.

Recently, many independent investigations have been performed to constrain the density dependence of the symmetry energy. A detailed summary of the recent progress can be found in Ref. [13] and the introduction of Ref. [14]. Lately, Agrawal *et al.* calculated the density distributions in both spherical and well deformed nuclei within microscopic framework with different energy density functionals giving $L = 59.0 \pm 13.0$ MeV [15]. Dong *et al.* probed the density dependence of the symmetry energy around the saturation den-

sity with the β^- -decay energies of odd- A heavy nuclei [14], and obtained $L = 50 \pm 15$ MeV. Wang and Li observed that a linear relationship between the L and the root-mean square (rms) charge radius difference of the ^{30}S - ^{30}Si mirror pair, and the estimated slope parameter is about $L = 54 \pm 19$ MeV from the coefficient of their proposed charge radius formula [16]. In this Brief Report, we employ the nuclear mass differences to derive the symmetry energy coefficient $a_{\text{sym}}(A)$ of heavy nuclei and then explore the density dependence of the nuclear matter symmetry energy at subsaturation densities.

We extract the $a_{\text{sym}}(A)$ with the differences of experimental nuclear mass [17] in order to reduce the uncertainties as far as possible. The binding energy $B(Z, A)$ of a nucleus can be described by the well-known liquid drop formula

$$B(Z, A) = a_v A - a_s A^{2/3} - E_c - a_{\text{sym}}(A)\beta^2 A + E_p + \dots \quad (1)$$

The Coulomb energy that includes charge exchange correction is given by

$$E_c = a_c \frac{Z(Z-1)}{A^{1/3}(1+\Delta)} (1 - 0.76Z^{-2/3}), \quad (2)$$

where the parameter Δ was introduced to describe the effect of the Coulomb interaction on the surface asymmetry and the effect of the surface diffuseness on the Coulomb energy [18], taking the form

$$\Delta = \frac{5\pi^2}{6} \frac{d^2}{r_0^2 A^{2/3}} - \frac{1}{1 + A^{1/3}/\kappa} \frac{N-Z}{6Z}. \quad (3)$$

$d \approx 0.55$ fm [18] is the diffuseness parameter in the Fermi function from the parametrization of nuclear charge distributions and r_0 is the nuclear-radius constant satisfying $3/(4\pi r_0^3) = 0.16\text{fm}^{-3}$. The meaning of the κ is discussed later. Here the independent variables are mass number A and isospin asymmetry β . Thus, the proton number is $Z = A(1 - \beta)/2$. The parameter $a_c = 0.71$ is known very well [19], in particular well determined from the masses of mirror nuclei [20, 21]. Performing a partial derivative of $B(Z, A)$ with respect to the

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isospin asymmetry β in Eq. (1), the symmetry energy coefficient can be expressed as

$$a_{\text{sym}}(A) = - \left(\frac{\partial B(Z, A)}{\partial \beta} + \frac{\partial E_c(Z, A)}{\partial \beta} \right) / (2\beta A). \quad (4)$$

Here the partial derivative $\partial B(Z, A)/\partial \beta$ is replaced by the difference

$$\frac{\partial B(Z, A)}{\partial \beta} \approx \frac{B(Z+1, A) - B(Z-1, A)}{\beta_2 - \beta_1}, \quad (5)$$

where β_1 and β_2 are the isospin asymmetry of nuclei $(Z-1, A)$ and $(Z+1, A)$, respectively. The difference of the Δ between the two neighboring nuclei is neglected since it is quite small. The nuclei involving magic numbers are excluded to avoid the strong shell effects. Because of the two neighboring nuclei $(Z-1, A)$ and $(Z+1, A)$ sharing the same oddity, the pairing energy is canceled out in binding energy difference $\Delta B = B(Z+1, A) - B(Z-1, A)$. This is one of the advantages of the present approach. The shell correction in the binding energy for the two neighboring nuclei should be close with each other because the densities of the energy levels are not expected to change distinctly considering they share the same oddity. Accordingly, the shell correction energies to their masses could be canceled to a large extent leading to a negligible correction to the ΔB . The contribution of the Coulomb energy is relatively clear, which is the primary advantage of this approach. This method should be better than that using the β^- -decay energies Q_{β^-} of odd-A heavy nuclei since the nuclear oddity changes in β -decay. On the other hand, much more experimental data are available in the present study compared to that using Q_{β^-} . The 168 experimental masses of $((Z-1, A), (Z+1, A))$ pairs of translead nuclei are used in the following analysis.

TABLE I: Comparison between the L values obtained in the present work and those from other recently independently analyses.

Reference	Method	L (MeV)
Ref. [30]	nuclear masses	$53 \lesssim L \lesssim 79$
Ref. [31]	quasiperiodic oscillation of SGR	$L \gtrsim 50$
Ref. [32]	empirical approach+density functionals	64 ± 5
Ref. [26]	FRDM-2011a	70 ± 15
Ref. [33]	giant quadrupole resonance energies	37 ± 18
Ref. [28]	pygmy dipole resonance	64.8 ± 15.7
Ref. [34]	alpha-decay energies	61 ± 22
Ref. [15]	empirical approach, density functionals	59.0 ± 13.0
Ref. [14]	beta-decay energies	50 ± 15
Ref. [16]	nuclear charge radius	54 ± 19
Ref. [35]	astrophysical observations of neutron star	$43 < L < 52$
Present	nuclear mass differences	50.0 ± 15.5

The mass dependence of the symmetry energy coefficient $a_{\text{sym}}(A)$ is given as [22]

$$a_{\text{sym}}(A) = \frac{S_0}{1 + \kappa A^{-1/3}}, \quad (6)$$

where κ is the ratio of the surface symmetry coefficient to the volume symmetry coefficient. Centelles *et al.* proposed a useful relation that the $a_{\text{sym}}(A)$ of finite nuclei is approximately

equal to $S(\rho_A)$ of the nuclear matter at a reference density ρ_A [23], which links the symmetry energy of the nuclear matter and the one of finite nuclei, and thus allows one to explore the density dependence of the symmetry energy $S(\rho)$. The previous calculations showed that the reference density $\rho_A \sim 0.55\rho_0$ for ^{208}Pb [14], where the model-dependence of the obtained ρ_A in units of ρ_0 is lowered greatly. The specific calculation process is similar to that in Ref. [14]. The formulism of DDM3Y shape in Ref. [24, 25] is applied to describe the density dependence of the symmetry energy $S(\rho)$

$$S(\rho) = 13.0 \left(\frac{\rho}{\rho_0} \right)^{2/3} + C_1 \left(\frac{\rho}{\rho_0} \right) + C_2 \left(\frac{\rho}{\rho_0} \right)^{5/3}. \quad (7)$$

This formula is much more universal than the usually used expressions $S(\rho) = S_0(\rho/\rho_0)^\gamma$ and $S(\rho) = 12.5(\rho/\rho_0)^{2/3} + C_p(\rho/\rho_0)^\gamma$ to describe the behavior of the symmetry energy around the saturation density as pointed out in Ref. [25], and it can provide both stiff and soft symmetry energy.

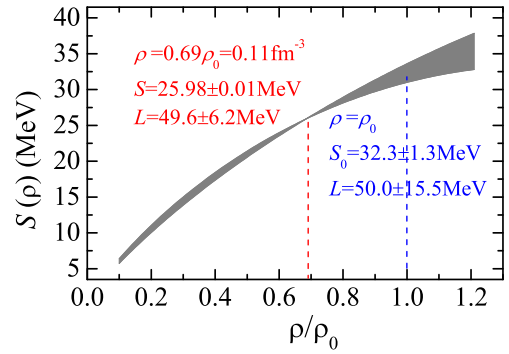


FIG. 1: (Color Online) Symmetry energy as a function of density.

Presently the symmetry energy S_0 at saturation density has been determined relatively well, we solely determine the optimal value of κ (carrying error bars) taking the S_0 as an input. The S_0 value has been constrained to rather narrow regions by some authors, such as $S_0 = 32.5 \pm 0.5$ MeV from the mass systematics [26], 32.10 ± 0.31 MeV from the double differences of experimental symmetry energies [27], 32.3 ± 1.3 MeV using the PDR analysis combined with the correlation between L and S_0 [28] and $31.5 - 33.5$ MeV from the calculations of half-infinite matter [29]. The S_0 from the Carbone *et al.* covering the other three is naturally believed to be the most acceptable one [14]. With their $S_0 = 32.3 \pm 1.3$ MeV as input, the calculated value of κ is $2.62^{+0.48}_{-0.46}$, thus the symmetry energy coefficient of ^{208}Pb is 22.4 ± 0.3 MeV. The parameters in Eq. (7) are $C_1 = 36.3 \mp 4.5$ MeV and $C_2 = -17.0 \pm 5.8$ MeV, and correspondingly the slope parameter of nuclear symmetry energy is $L = 50.0 \pm 15.5$ MeV. Incidentally, if the density dependent behavior $S(\rho) = S_0(\rho/\rho_0)^\gamma$ is applied, one obtains $L = 59.3 \pm 6.7$ MeV. The reference density ρ_A plays an important role to determine the slope parameter L . If the widely used $\rho_A = 0.1 \text{ fm}^{-3}$ for ^{208}Pb and $\rho_0 = 0.16 \text{ fm}^{-3}$ are employed, the obtained slope parameter is $L = 74.0 \pm 17.5$ MeV.

These overestimate the L value compared with the present calculations. Table I shows the present estimated L values compared with those from other approaches. One can see clearly that the present finding has a remarkable overlap with these results, in particular it is very consistent with the later results, indicating that one may relatively well understand the density-dependent behavior of symmetry energy around the saturation density. The present approach is much more straightforward than those applying the binding energy directly. In this work, the effect of the Coulomb interaction on the surface asymmetry and the effect of the surface diffuseness on the Coulomb energy that described by Δ in Eq. (1) are taken into account, which tend to be neglected in many previous investigations. If we neglect these two effect, the estimated $a_{\text{sym}}(A)$ of ^{208}Pb should be 22.9 ± 0.3 MeV, and correspondingly the slope parameter is $L = 44.0^{+14.8}_{-15.6}$ MeV. Therefore, these effects cannot be discarded optionally. Fig. 1 displays the density dependent behavior of the symmetry energy versus density. It has been shown that the neutron skin thickness of heavy nuclei is uniquely fixed by the symmetry energy density slope $L(\rho)$ at a subsaturation cross density $\rho \approx 0.11\text{fm}^{-3}$ rather than at saturation density ρ_0 [36]. And the giant monopole resonance of heavy nuclei has been shown to be constrained by the EOS of nuclear matter at $\rho \approx 0.11\text{fm}^{-3}$ rather than at saturation density [37]. Most interestingly, we find that at the density of $\rho = 0.69\rho_0 = 0.11\text{fm}^{-3}$, the symmetry energy is $S(\rho = 0.11\text{fm}^{-3}) = 25.98 \pm 0.01$ MeV, being agreement with 26.2 ± 1.0 MeV that from the Skyrme forces in Ref. [19] and 26.65 ± 0.20 MeV using data on neutron skin thickness of Sn isotopes and binding energy differences for a number of heavy isotope pairs [36]. The small error bar results from the fact that different curves almost intersect at this point as shown in Fig. 1. Incidentally, if one use the reference density $\rho_A = 0.1\text{fm}^{-3}$ for ^{208}Pb , the obtained symmetry energy

is $S(\rho = 0.11\text{fm}^{-3}) = 24.2 \pm 0.1$ MeV, which is lower than that in the present calculation and in Ref. [19, 36]. This further suggests the importance of obtaining an accurate reference density. In addition, the slope parameter is estimated to be $L = 49.6 \pm 6.2$ MeV, consistent with $L = 46.0 \pm 4.5$ MeV in Ref. [36]. These results at $\rho \approx 0.11\text{fm}^{-3}$ are perhaps useful to determine the neutron skin thickness with a higher accuracy and to explore the cooling of canonical neutron stars.

In summary, the symmetry energy coefficients of heavy nuclei were determined with the available experimental nuclear masses of heavy nuclei. This approach prevents interferences from other energy terms very effectively. The calculated symmetry energy coefficient of ^{208}Pb was furthermore employed to probe the density-dependent symmetry energy of nuclear matter. With the symmetry energy $S_0 = 32.3 \pm 1.3$ MeV at saturation density in Ref. [28] as an input, the estimated values of the slope parameter is $L = 50.0 \pm 15.5$ MeV, which agrees with very recent results, such as these from the rms charge radius difference of the mirror nuclei [16], the β^- -decay energies of odd-A heavy nuclei [14] and astrophysical observations of neutron star [35]. Moreover, we pay special attention to the symmetry energy at the density of $\rho = 0.69\rho_0 = 0.11\text{fm}^{-3}$ due to its importance. The symmetry energy and the correspondingly slope parameter are $S(\rho = 0.11\text{fm}^{-3}) = 25.98 \pm 0.01$ MeV and $L = 49.6 \pm 6.2$ MeV respectively, which are consistent with few published results. To reduce the uncertainty of the L value, one need to reduce the uncertainty of the S_0 value as far as possible.

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